On the Representation of a Solution for the Perturbed Quasi-Linear Controlled Neutral Functional Differential Equation

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The neutral functional differential equation is a mathematical model of such system whose behavior at a given moment depends on the velocity and state of the system in the past. Many real processes are described by neutral functional differential equations and the theory of such equations is presented in [1-3]. In this work, for the controlled neutral functional differential equation

\[ \dot{x}(t) = A(t, x(t), x(t-h))\dot{x}(t-\sigma) + f(t, x(t), x(t-\tau), u(t)), \quad t \in [t_0, t_1] \]

with the initial condition

\[ x(t) = \varphi(t), \quad t < t_0, \quad x(t_0) = x_0 \]

we prove the analytic representation formula of a solution – Variation formula of solution, which is obtained in the left neighborhood of the endpoint of the main interval. In the formula, the effects of perturbations of the delay parameters \( h, \sigma, \tau \), the initial vector \( x_0 \), the initial \( \varphi(t) \) and control \( u(t) \) functions are detected.

In the particular case, when the matrix function \( A(\cdot) \equiv 0 \), Variation formulas of solutions for different type of delay functional differential equations are proved in [4,5]. Theorems of such a type play an important role in studying optimal control problems, proving of the necessary conditions of optimality.


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